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EVALUATION OF MEASUREMENT QUALITY USING THE MONTE-CARLO METHOD

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ABSTRACT

The article discusses the dependence of measurement value the uncertainty on the number of experiments performed under different confidence probability conditions. The computer experiment was performed in the LabVIEW software environment. Theoretical and experimental research has shown that the quality of measurement depends not only on the probability of confidence and number of experiments conducted, but also on the type of law of probability distribution of value. The measurement quality is much higher for a normally distributed quantity, while the lowest quality is obtained for a uniform distributed value.

АННОТАЦИЯ

В статье обсуждается зависимость величины неопределенности измерения от количества экспериментов, проведенных при различных условиях доверительной вероятности. Компьютерный эксперимент проводился в программной среде LabVIEW. Теоретические и экспериментальные исследования показали, что качество измерения зависит не только от вероятности достоверности и количества проведенных экспериментов, но и от типа закона распределения вероятностей значений. Качество измерения намного выше для нормально распределенной величины, в то время как самое низкое качество получается для равномерно распределенной величины.

Keywords: Monte-Carlo Method, measuring, LabVIEW, non-ordinary.

Ключевые слова: метод Монте-Карло, измерения, LabVIEW, неосновной.

Introduction

In any field of science, technology and modern life it is difficult to name a field, process, event that is not valued by quality. Obviously, the quality of the measurement is no exception.

Although numerous articles and studies [1-5] have been devoted to measuring the quality of measurement, in fact quantitative evaluation is obtained only when the measurable quantity is distributed according to the law of normal distribution. In the case where the law of measurement parameter distribution is not close to normal, determining the quality of the measurement is a task that requires radical additional research. Our era is characterized by rapidly evolving new technologies that are radically changing human activities and lives. One such new and revolutionary technology is virtual device technology, which imitates real physical devices. The LabVIEW software environment is a clear example of such technologies.

The main purpose of the article is to theoretically and experimentally determine the dependence of the uncertainty of the point evaluation of measurable value on the number of experiment performed (not exceeding 20), under different confidence probability conditions (P = 0.8; 0.9; 0.95; 0.99). The study should be conducted both for the normally distributed X value and for the case where the X-value is subject to the laws of uniform and Laplace distribution. Computer implementation involve the use of the LabVIEW software environment.

1.1. The main part.

Accuracy of Mathematical Expectation Estimation According to the Monte-Carlo method

As it is known, measurement quality is a set of measurement properties that ensure the compliance of the means, method, methodology, measurement conditions and unity with the requirements that ensure the solution of the measurement task. Therefore, the quality of the measurement refers to the set of features that determine the accuracy of a given measurement accuracy given in a given form and performed in a given time. Measurement quality indicators are: accuracy, correctness and truthfulness. [1-3]

When using the statistical model of the measurement process, the measurable value is determined by the function of density probability distribution and numeral characteristics: mathematical expectation, dispersion, mean square deviation, correlation coefficient, etc. But because the number of measurements or observations is always limited, we are forced to be satisfied with estimates of these numerical characteristics only. Evaluation must be thorough, irreplaceable, effective [2]. These requirements are met by the arithmetic mean of N observations.

Task formulation: Assume that an observation of the N number of observation on random values X is made. The mathematical expectation of X is \( m_x \) and the mean square deviation is \( \sigma_x \). On the basis of n measurements is obtained \( m_x^* \) - mathematical expectation estimation. If n is large enough, determine P probability that the estimate differs from \( m_x \) true value by not more than \( \varepsilon \) > 0 values. \( \varepsilon \) means the uncertainty of the point estimate and determines the accuracy of the estimate [2-5].

The solution to this task is considered in three cases:
1) X random value has a law of normal distribution;
2) The law of distribution of value X is unknown.
3) The length X has a symmetrical, unimodal distribution.

1. It is known that in the first case the probability of P-confidence is given.

\[
|m_x^* - m_x| \leq \varepsilon_0 = \frac{\sigma \cdot t}{\sqrt{n}}
\]  

(1)

\( t \) is a parameter that depends on P-confidence probability and is given in the table.

1. According to the (1), the minimum value of n number of experiments, that will satisfy the value of \( \varepsilon \), can be determined.

\[
n = \left( \frac{\sigma}{\varepsilon} \right)^2
\]  

(2)

2. When the law of distribution of the value x is unknown, it is assumed that in this case the value must be assigned the law of uniform distribution (all values are equally expected)

\[
\varepsilon = \frac{\sigma}{\sqrt{n(1-P\varepsilon)}}
\]  

(3)

Therefore:

\[
n = \left( \frac{\sigma}{(1-P\varepsilon)^{3/2}} \right)^2
\]  

(4)

3. If the density of the distribution of x random values decreases symmetrically on both sides of the mathematical expectation, then

\[
\varepsilon = \frac{2}{3} \frac{\sigma}{\sqrt{n(1-P\varepsilon)}}
\]  

(5)

From where

\[
n = \left( \frac{2}{3 \sqrt{n(1-P\varepsilon)\varepsilon}} \right)^2
\]  

(6)

These are the expressive use in the compiling an algorithm and performing an experiment in the LabVIEW software environment.

The case where x random quantity is distributed according to the law of normal distribution is particular importance, because in the case of a very large number of experiments the value tends to return to normal. For small numbers of experiments the random x value is described by the Student distribution [2], which depends only on the number of experiments performed. There are statistical tables according to which the value of parameter t is found when the degree of freedom is equal to n-1.
Experimental Part.
1.2. Determining the Uncertainty Band in the Case of a Normal Distribution of the Value X

We have already considered the case where the measurable quantity is subject to the law of normal distribution. It is known from statistics that the law of Student distribution can describe the law of normal distribution if the number of measurements does not exceed 20.

Figure 1 shows a block diagram of a program created in the LabVIEW environment. At the entrance to the scheme is provided the realization of 20 random numbers with a student distribution, whose mathematical expectation is equal to 2 and the mean square deviation is 0.5.

The value of the uncertainty band (width) is obtained at the output of the scheme, which determines the reliable intervals (interval1, interval2).

The results of experiment are given in Table 1, and the corresponding graphs are given in figure 2. The graph shows the dependence of the value of the uncertainty band on the number of experiments performed.

Table 1.

<table>
<thead>
<tr>
<th>n</th>
<th>width = Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.8</td>
</tr>
<tr>
<td>9</td>
<td>0.9</td>
</tr>
<tr>
<td>0.55</td>
<td>0.55</td>
</tr>
<tr>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>5</td>
<td>0.38</td>
</tr>
<tr>
<td>7</td>
<td>0.25</td>
</tr>
<tr>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>10</td>
<td>0.26</td>
</tr>
<tr>
<td>0.34</td>
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<tr>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>15</td>
<td>0.21</td>
</tr>
<tr>
<td>0.28</td>
<td>0.28</td>
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<tr>
<td>0.34</td>
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<tr>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>0.36</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Figure 1. Block Diagram

Figure 2. The dependence of the value of non-ordinary band of the number of experiments (Students distribution)
Different curves correspond to different values of confidence probability.

The first – 0.8; Second – 0.9; The third – 0.95 and the fourth – 0.99.

Figure 3 shows the Front Panel, which is a theoretically derived dependence of the indefinite $\varepsilon$ strip on “n” points, where $\varepsilon = \frac{\sigma}{\sqrt{n}}$.

Analysis of the experimental and theoretically obtained results shows that the obtained results and the theoretically existing $\varepsilon(n)$ dependence have the following character: In both cases, there is a decreasing nature of the dependence, only the experimental shows some different meanings. This must be so because in the same formula, different mean values of the mean square deviation are obtained at different “n”s. In addition, the value of the selection starts from zero, which is less than expected.

Determining the uncertainty band of X value in the case of Laplace distribution

We conducted a similar experiment for the Laplace distribution law, which belongs to the law of unimodal, and according to the Monte-Carlo method, in this case we should use the following effigy: $\varepsilon = \frac{2}{\sqrt{n(1-p)}} \frac{\sigma}{\sqrt{n}}$

The results of the experiment are shown in Table 2.

Table 2.

<table>
<thead>
<tr>
<th>n</th>
<th>p</th>
<th>width = $\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.8</td>
<td>0.67</td>
</tr>
<tr>
<td>7</td>
<td>0.9</td>
<td>0.94</td>
</tr>
<tr>
<td>10</td>
<td>0.95</td>
<td>1.33</td>
</tr>
<tr>
<td>15</td>
<td>0.99</td>
<td>2.98</td>
</tr>
<tr>
<td>20</td>
<td>0.8</td>
<td>1.49</td>
</tr>
</tbody>
</table>

Laplacian distribution
It should be noted that the results obtained for the estimation of different $x$’s are again reflected in similar curves, which also confirm the decrease of $\varepsilon$ with increasing $n$.

**Determining the uncertainty band of $X$ value in case of equal distribution**

An experiment was performed when the law of distribution of value $x$ is unknown. In this case it is advisable to use the law of uniform distribution as stated above.

The results of the experiment are given in Table 3 and Figure 5 is a graphical visualization of the results.

**Table 3.**

<table>
<thead>
<tr>
<th>$n$</th>
<th>$p$</th>
<th>width $= \Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.8</td>
</tr>
<tr>
<td>5</td>
<td>2.36331</td>
<td>3.34223</td>
</tr>
<tr>
<td>7</td>
<td>1.99736</td>
<td>2.6247</td>
</tr>
<tr>
<td>10</td>
<td>1.67111</td>
<td>2.36331</td>
</tr>
<tr>
<td>15</td>
<td>1.36446</td>
<td>1.92964</td>
</tr>
<tr>
<td>20</td>
<td>1.18156</td>
<td>1.67111</td>
</tr>
</tbody>
</table>

**Figure 4. The dependence of the value of non-ordinary band of the number of experiments (Laplace distribution)**

**Figure 5. The dependence of the value of non-ordinary band of the number of experiments (The uniform distribution)**
1.3. Quantitative comparison of results

Quantitative comparisons of the obtained results are shown in Table 4 and 5.

Table 4.

<table>
<thead>
<tr>
<th>P</th>
<th>s</th>
<th>The law of normal distribution</th>
<th>The law of equal distribution</th>
<th>The law of Laplace distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>1.30</td>
<td>0.393083</td>
<td>1.0987</td>
<td>0.732467</td>
</tr>
<tr>
<td>0.9</td>
<td>1.70</td>
<td>0.578286</td>
<td>2.03189</td>
<td>1.35459</td>
</tr>
<tr>
<td>0.95</td>
<td>2</td>
<td>0.718132</td>
<td>3.38062</td>
<td>2.37566</td>
</tr>
<tr>
<td>0.99</td>
<td>2.75</td>
<td>1.02901</td>
<td>10.394</td>
<td>6.92935</td>
</tr>
</tbody>
</table>

Table 5.

<table>
<thead>
<tr>
<th>P</th>
<th>s</th>
<th>The law of normal distribution</th>
<th>The law of equal distribution</th>
<th>The law of Laplace distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>1.30</td>
<td>0.328877</td>
<td>0.919239</td>
<td>0.612826</td>
</tr>
<tr>
<td>0.9</td>
<td>1.70</td>
<td>0.483828</td>
<td>1.7</td>
<td>1.13333</td>
</tr>
<tr>
<td>0.95</td>
<td>2</td>
<td>0.600833</td>
<td>2.82843</td>
<td>1.98762</td>
</tr>
<tr>
<td>0.99</td>
<td>2.75</td>
<td>0.86093</td>
<td>8.69626</td>
<td>5.79751</td>
</tr>
</tbody>
</table>

Analysis of these tables shows that when n=7 and n=10 uncertainty bands increase with increasing confidence probability, the law of all three distributions.

2. Conclusion

Based on the analysis of the obtained results, we can draw the following conclusions:

1. As the number of \( n \) experiments increases, the uncertainty of the point estimation (arithmetic mean of \( x \)) decreases for all values of the confidence probability.

2. The higher the probability of confidence, the greater the value of the corresponding uncertainty band \( \epsilon \).

3. The amplitude of the uncertainty \( \epsilon \) depends on the type of the law of distribution of value: the best result is obtained for the case of the normal distribution of the value \( x \), (we use the law of distribution of the student).

The analysis of the obtained results shows that the measurement quality is much higher for the normally distributed value, while the lowest quality has a uniform distributed value.

References: